

If both t_1 and t_2 are much larger than T_N , Eq. 5-50 reduces to

$$\frac{\frac{C_1 B}{R_m}}{1 - \frac{1}{2\mu}} + \frac{\frac{C_2 B}{R_m}}{1 - \frac{1}{2\mu}} = 1 \quad (5-51)$$

If both t_1 and t_2 are much smaller than T_N , Eq. 5-50 reduces to

$$\frac{\frac{C_1 B}{R_m}}{\frac{T_N}{\pi t_1} \sqrt{2\mu-1}} + \frac{\frac{C_2 B}{R_m}}{\frac{T_N}{\pi t_2} \sqrt{2\mu-1}} = 1 \quad (5-52)$$

The maximum error in this procedure occurs when an extremely short pulse is combined with an infinitely long one (see Fig. 5-14). For this case, a better approximation is obtained if the sum of the kinetic energy imparted by the impulse and the work done by the quasi-static pressure is equated to the strain energy in the system, i.e.,

$$\frac{1}{2M} \left[\frac{C_1 B t_1}{2} \right]^2 + C_2 B X_m = R_m X_m - \frac{R_m X_e}{2} \quad (5-53)$$

Making the same substitutions as were made for Eq. 5-43 and rearranging terms, it is found that

$$\left[\frac{\frac{C_1 B/R_m}{\frac{T_N}{\pi t_1} \sqrt{2\mu-1}}}{1 - \frac{1}{2\mu}} \right]^2 + \frac{C_2 B/R_m}{1 - \frac{1}{2\mu}} = 1 \quad (5-54)$$

where the subscript 1 refers to the impulse component and all other terms are as previously defined.

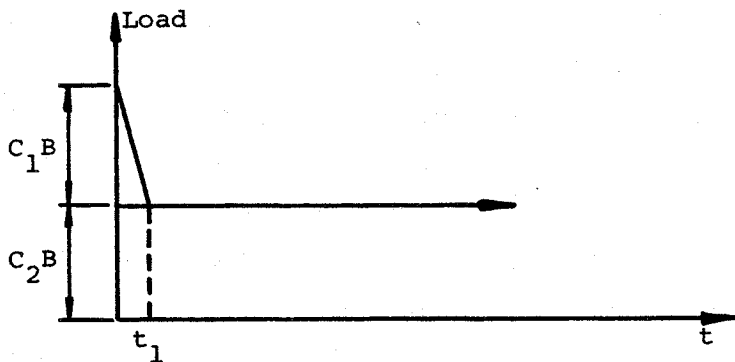


Figure 5-14. Impulsive Load Plus Long Duration Component

Although Eq. 5-54 is derived for a constant amplitude second pulse, it can be applied to other loadings where the duration of the initial pulse is less than one-fifth the period and the duration of the second pulse is greater than 10 times the period of the system. Alternatively, Eq. 5-55 can be used for those loadings where the initial pulse is less than one-fifth the period and the second pulse has a long, but finite duration.

$$\left[\frac{\frac{C_1 B}{R_m}}{\frac{T_N}{\pi t_1} \sqrt{2\mu - 1}} \right]^2 + \frac{\frac{C_2 B}{R_m}}{\frac{T_N}{\pi t_2} \sqrt{2\mu - 1} + \frac{1 - \frac{1}{2\mu}}{1 + 0.7 \frac{T_N}{t_2}}} = 1 \quad (5-55)$$

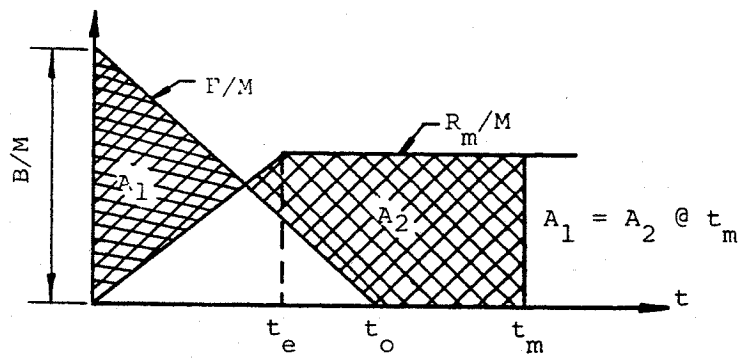
Appendix B includes charts of Eq. 5-54 which can be used for preliminary design or analysis.

The equations of this section and response charts of Appendix B can be used in two ways. If the properties of the system and load characteristics are specified, the maximum response can be obtained directly in terms of the ductility ratio, μ . For a single pulse approximation of the loading, the determination of response is straightforward. For multiple triangle approximations of the loading, it will be necessary to either assume various values of μ until the appropriate equation from Eqs. 5-49 through 5-55 is satisfied or to solve the equation for μ .

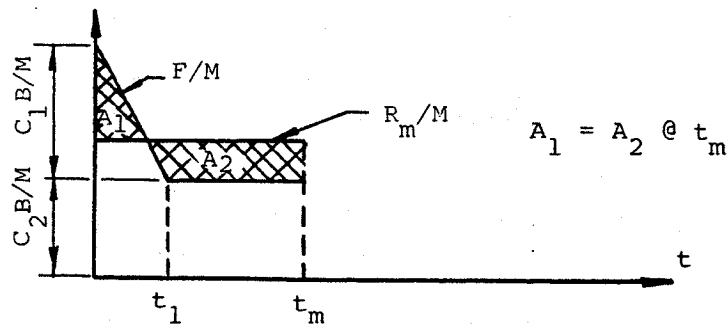
If only the load characteristics and a desired maximum response are specified, an iterative process is required for design. In order to start the process, it is necessary to assume some trial section properties. With these properties, the system can be designed and its maximum response obtained. If the required resistance is more than the trial section resistance, the process must be repeated until the required resistance is equal to or less than the trial section resistance.

Equations 5-37, 5-43, 5-44, and 5-47 through 5-55 are valid only if response extends beyond the elastic limit, i.e., $\mu \geq 1$. Inspection of these equations shows the benefit of allowing inelastic response to take place. Equation 5-37 indicates that, if a large value of μ is permitted, the required resistance of the element may be equal to B . For elastic response ($\mu=1$) the resistance must be equal to $2B$. Equation 5-44 shows that for the elastic case ($\mu=1$), the required resistance is equal to $i\omega_N$, but the required resistance approaches zero as μ increases. Note also that the load term " B " and the resistance term " R_m " may be either total load and total resistance (B, R_m) or unit load and unit resistance (P_r, r_m).

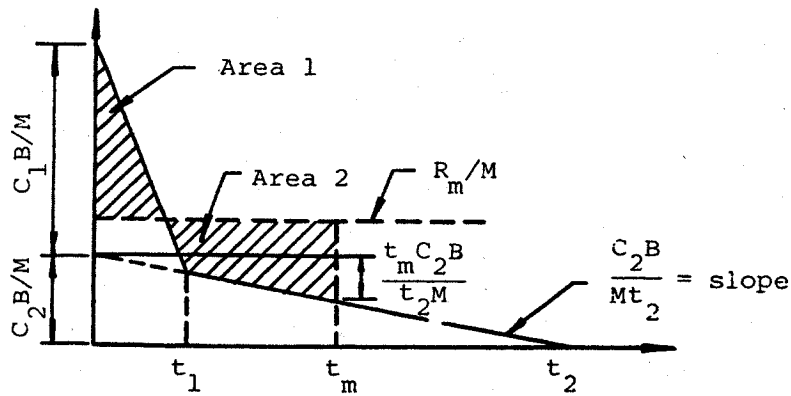
It is sometimes important to determine the time of maximum response of the structural element. If the ordinates of the loading and resistance functions are divided by the mass, M , they can be plotted as shown in Fig. 5-15. The loading can be considered an acceleration function and the resistance a deceleration function. The area under either plot is equal to the velocity change of the system due to the loading or resistance of the element. Since the velocity of the mass is equal to zero at t_m , the areas under the loading and resistance plots must be equal at this time. If one diagram is subtracted from the other, the net area must be equal to zero. If one area is subtracted from the other as shown in Fig. 5-15, then A_1 must equal A_2 . Although the concept is simple, a difficulty arises in determining the time, t_e , to reach maximum resistance. An iterative technique, such as numerical integration, can be used to determine t_e . Alternatively, a rigid-plastic resistance, i.e., $t_e=0$, might be assumed to obtain an approximation of t_m . The percent error in such an approximation decreases with increasing μ . Figure 5-15(b)



(a) Triangular Load



(b) Impulse Plus Quasi-Static Load with Rigid-Plastic Resistance



(c) Double Pulse Loading with Rigid-Plastic Resistance

Figure 5-15. Acceleration Versus Time Plots

demonstrates how this technique might be applied to an impulsive load superimposed on a quasi-static type loading and Fig. 5-15(c) applies the technique to a two-triangle representation of the airblast loading. Assuming a rigid-plastic resistance function, the areas under the loading and resistance plots in Fig. 5-15(b) will be equal at t_m , i.e.,

$$\frac{C_1 B t_1}{2M} + \frac{C_2 B t_m}{M} = \frac{R_m t_m}{M} \quad (5-56)$$

or

$$t_m = \frac{C_1 B t_1}{2(R_m - C_2 B)} \quad (5-57)$$

Assuming a rigid-plastic resistance function for the two triangle loading shown in Fig. 5-15(c), the areas under the loading and resistance functions can again be equated, i.e.,

$$\frac{C_1 B t_1}{2M} + \left[C_2 B - \frac{C_2 B t_m}{2t_2} \right] \frac{t_m}{M} = \frac{R_m t_m}{M} \quad (5-58)$$

Expanding Eq. 5-58 and solving for t_m , it is found that

$$t_m = \frac{C_2 B - R_m \pm \left[(R_m - C_2 B)^2 + \frac{(C_2 B t_1)(C_1 B t_1)}{t_2} \right]^{0.5}}{C_2 B} \quad (5-59)$$

5.5.3 Numerical Integration

This analytical technique obtains the response of the system by numerical integration of the differential equation of motion. It is the most general and versatile method of analysis for many problems of interest. It can be applied to any system with a finite number of degrees of freedom and can treat any force-displacement-time relationship, ranging

from linear elastic to nonlinear, viscoelastic-plastic relations. Numerical integration has found wide application on electronic computing devices for compiling the solutions to simple problems, and for the rapid solution of problems in the dynamics of complicated systems. For hand computation, the method is best suited to systems of a few degrees of freedom with simple force-resistance relations, such as the bilinear elastic or elastic-plastic resistances.

Rewriting Eq. 5-30 in the form

$$\ddot{X} = \frac{F(t) - KX}{K_{LM}M_t} \quad (5-60)$$

it is seen that if $F(t)$ and X are known at any particular instant of time, the acceleration of the mass, M , can be calculated. The basis of the method of numerical integration is the subdivision of time into intervals, Δt , and an assumption of the nature of the variation of the acceleration during the time interval. The procedure recommended herein is presented in Refs. 5-15 and 5-16. It is convenient to adopt the notation developed in Ref. 5-15. If \ddot{X}_n , \dot{X}_n , X_n , are the acceleration, velocity and displacement, respectively, at time $t = t_n$, then the velocity and displacement of the mass at time $t = t_n + \Delta t$ are given by

$$\dot{X}_{n+\Delta t} = \dot{X}_n + \frac{1}{2} \Delta t (\ddot{X}_n + \ddot{X}_{n+\Delta t}) \quad (5-61)$$

$$X_{n+\Delta t} = X_n + \Delta t \dot{X}_n + \frac{(\Delta t)^2}{2} \ddot{X}_n + \beta (\ddot{X}_{n+\Delta t} - \ddot{X}_n) (\Delta t)^2 \quad (5-62)$$

If the variation of the acceleration over the time interval Δt is linear, β is taken equal to $1/6$. If a constant acceleration equal to the average of \ddot{X}_n and $\ddot{X}_{n+\Delta t}$ is assumed over the time interval, β is taken equal to $1/4$. Values of β of 0 and $1/12$ can also be given simple geometric interpretations.

The method proceeds as follows. The acceleration, velocity, and displacement at $t = 0$ are computed or obtained from the given initial conditions. Then for $t = \Delta t$, the

acceleration $\ddot{X}_{n+\Delta t}$ is assumed. Using Eqs. 5-61 and 5-62, the velocity and displacement at time $t_{n+\Delta t}$ are computed. Knowing the displacement $X_{n+\Delta t}$, the resistance KX can be evaluated. This value is then substituted into Eq. 5-60 and the assumed acceleration checked. If the assumed and resultant acceleration are not in agreement, the computed acceleration is used for the next trial and the computational process repeated until the procedure converges to the correct solution. When convergence is obtained, the next time increment is added and the process repeated.

The criteria which are important in the application of numerical integration are convergence, rate of convergence, stability, length of time interval, and choice of β . All of these criteria are interrelated and have been studied fairly extensively. The stability and convergence criteria for an undamped single degree of freedom system will generally be satisfied if the ratio $\Delta t/T_N$ is less than about 0.2. For systems with several degrees of freedom, the stability and convergence limits must be applied in terms of the natural period of the highest mode of vibration, i.e., the minimum natural period. The choice of a time interval also determines the number of iterations required to properly describe structural response and, therefore, affects the cost of the analysis in terms of computer time. Another consideration is that the time interval should be small enough to adequately describe the time variation of the forcing function.

The choice of β governs the accuracy and ease of application of the method. Extensive work in the application of this method has resulted in the following conclusions. A $\beta = 1/6$ is best suited for forced vibrations of systems with damping and with initial velocity and displacement. The best results in amplitude of response for an undamped system are obtained using $\beta = 1/4$. A $\beta = 1/12$ gives the most rapid and accurate results for an undamped system without initial

velocity. For very rapid results, where accuracy is not of primary importance, $\beta = 0$ often proves useful.

5.5.4 Spherical Chambers

Spherical chambers are used for some suppressive shield applications where the fragment hazards are minimal. If it is assumed that the sphere responds only in the fundamental mode, it can be analyzed as a single degree of freedom system using the techniques described earlier in this chapter. Its elastic period of vibration is given by Eq. 5-24. Its static resistance can be obtained from Eq. 5-17 or 5-21. Illustrative example 5.6.4 uses this approach to analyze a Group 6A shield.

Reference 5-12 proposes an approximate expression for the maximum stress in a spherical suppressive shield. It is based on computer solutions of the differential equation of motion for the sphere and applies only to the elastic case.

Reference 5-17 offers closed form solutions to the elastic-plastic response of thin spherical shells to internal blast loading. In order for results to be obtained, solutions to non-linear differential equations are required.

Reference 5-20 uses an energy method to obtain the plastic response; however, this report does not consider the effects of impulse and quasistatic pressure simultaneously.

For additional design information, references 5-21 and 5-22 discuss spherical chamber component parts, fabrication techniques, and test results for the Group 6A and 6B shields.

5.6 ILLUSTRATIVE EXAMPLES

5.6.1 Response of the Group 3 Suppressive Shield Walla. Given

A 48.8 pound charge of Pentolite is detonated inside the Group 3 Suppressive Shield. The Group 3 Shield is a cylindrical structure with a flat reinforced concrete roof and floor. The cylindrical body of the structure is fabricated from 296 interlocking S3x5.7 I-beams. The inner layer of I-beams has an inside radius of 5 feet 7.5 inches. The inside height of the structure is 10 feet. A layered steel reinforcing ring is placed around the outer circumference of the body at a distance of 5 feet above the floor.

b. Find

The maximum response of beam elements in the wall of the shield to airblast loading.

c. Solution

The first step is to determine the blast loading seen by the wall. The TNT equivalent for Pentolite is given in Table 3-1 as 1.129. The equivalent charge weight of TNT from Eq. 3-1, pg. 3-4 is

$$W = 1.129(48.8) = 55.1 \text{ lb TNT}$$

and

$$W^{1/3} = 3.805 \text{ lb}^{1/3}$$

The scaled distance from the charge to the mid-height of the wall from Eq. 3-2, pg. 3-6 is

$$Z = \frac{R}{W^{1/3}} = \frac{5.625}{3.805} = 1.478 \text{ ft/lb}^{1/3}$$

Values of peak reflected pressure and scaled specific impulse as a function of scaled distance are plotted in Fig. 3-6. For $Z = 1.478 \text{ ft/lb}^{1/3}$

$$P_r = 3350 \text{ psi}$$

and

$$\frac{i_r}{W^{1/3}} = 0.111 \text{ psi-sec/lb}^{1/3}$$

or

$$i_r = 0.422 \text{ psi-sec}$$

The duration of the reflected pulse is obtained from Eq. 3-4, pg. 3-14.

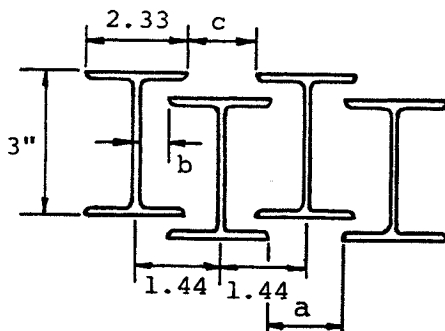
$$t_r = 2i_r/P_r = 0.00025 \text{ sec}$$

The peak quasi-static pressure is found from Fig. 3-9. For

$$\frac{W}{V} = \frac{55.1}{\pi (5.625)^2 (10)} = 0.0554 \text{ lb/ft}^3$$

$$P_{qs_{\max}} = 187 \text{ psi}$$

To determine the duration of the quasi-static pressure, the vent area ratio, α_e , of the structure must be found. Only the cylindrical wall provides venting. An idealized representation of the interlocking I-beams which make up the wall is shown below. Referring to Fig. 3-7(d),



Group 3 Shield Wall Section

$$a = c = 2 \left(1.44 - \frac{2.33}{2} \right) = 0.55 \text{ inch}$$

$$b = 1.44 - \frac{2.33}{2} - 0.5(0.19) = 0.179 \text{ inch}$$

Then,

$$A_{v_1} = 2 \ln a_i = 2(120)(148)(0.55) = 19,358.4 \text{ in}^2$$

$$A_{v_2} = A_{v_3} = 2 \ln b_i = 2(120)(148)(0.179) = 6358.1 \text{ in}^2$$

$$A_{v_4} = 2 \ln c_i = 2(120)(148)(0.55) = 19,358.4 \text{ in}^2$$

$$A_w = nL(2.33 + 0.545) = 148(120)(2.875) = 51,060 \text{ in}^2$$

and

$$\alpha_1 = \alpha_4 = \frac{A_{v_1}}{A_w} = \frac{19,358.4}{51,060} = 0.3791$$

$$\alpha_2 = \alpha_3 = \frac{A_{v_2}}{A_w} = \frac{6358.1}{51,060} = 0.1245$$

From Eq. 3-5, pg. 3-15,

$$\frac{1}{\alpha_e} = \sum_{i=1}^n \frac{1}{\alpha_i} = \frac{1}{0.3791} + \frac{1}{0.1245} + \frac{1}{0.1245} + \frac{1}{0.3791} = 21.34$$

or

$$\alpha_e = 0.0469$$

The scaled maximum pressure is

$$\bar{p} = \frac{P_{qs} + P_o}{P_o} = \frac{187 + 14.7}{14.7} = 13.72$$

The interior surface area of the shield wall is

$$A_i = 2\pi rh = 2(3.14)(5.625)(10) = 353.43 \text{ ft}^2$$

From Fig. 3-10 for a scaled maximum pressure of 13.72, the scaled blowdown time is

$$t_b a_o \alpha_e A_i / V = 1.23$$

Substituting the known parameters

V = volume of structure (994 ft³)

α_e = vent area ratio (0.0469)

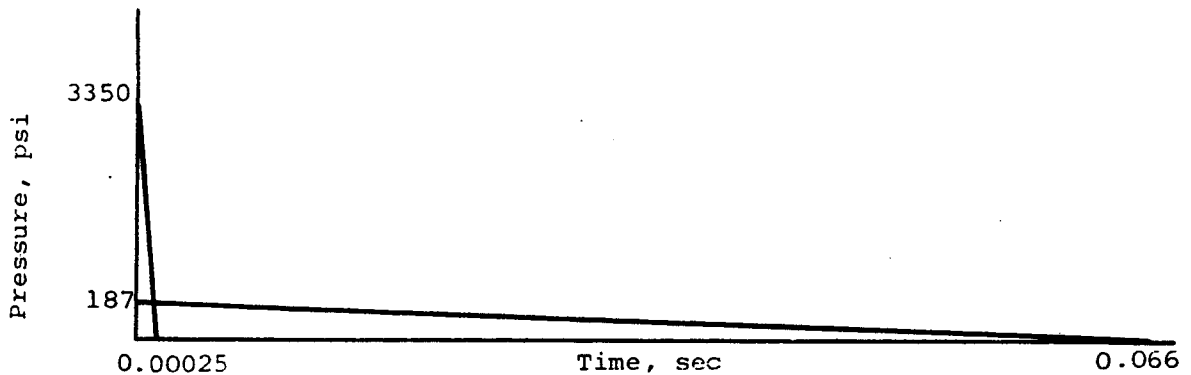
A_i = surface area of cylinder wall (353.43 ft²)

a_o = sound velocity in air (1117 ft/sec)

and solving for t_b , it is found that the blowdown time is

$$t_b = 0.066 \text{ sec}$$

The blast pressure loading on the wall of the structure is approximated by a double triangular pressure pulse as shown below.



Blast Loading of Group 3 Shield I-Beam

$$C_1 = \frac{3350 - 187}{3350} = 0.944$$

$$C_2 = \frac{187}{3350} = 0.056$$

The structure responds dynamically to both the reflected pressure pulse and the quasi-static pressure. Response of the wall is determined by considering an individual I-beam. The beam is assumed to have fixed ends and a span of 5 feet. Because of the arrangement

of the interlocking I-beams, the effective width over which the blast load acts is assumed to be 1.44 inches per beam. (See previous sketch of Group 3 Shield wall section).

The section and material properties of the S3x5.7 beam are (Ref. 5-2)

$$\begin{aligned} w &= 5.7 \text{ lb/ft} & E &= 29 \times 10^6 \text{ psi} \\ I &= 2.52 \text{ in}^4 & f_y &= 36,000 \text{ psi} \\ S &= 1.68 \text{ in}^3 & f_{dy} &= 39,600 \text{ psi (Table 4-1)} \\ L &= 5 \text{ ft} = 60 \text{ inches} \end{aligned}$$

The natural period of a beam fixed at both ends is obtained from the expression for natural frequency in Fig. 5-8, i.e.,

$$T_N = \frac{2\pi}{\omega_N} = 0.28L^2 \sqrt{w/EIg}$$

where

$$w = \text{weight, lb/in} = 0.475 \text{ lb/in}$$

$$g = \text{gravitational acceleration, in/sec}^2 = 386 \text{ in/sec}^2$$

For the beam only, the natural period is

$$\begin{aligned} T_N &= 0.28(60)^2 \sqrt{0.475/[29(10)^6(2.52)(386)]} \\ &= 0.00414 \text{ sec} \end{aligned}$$

The maximum resistance of a uniformly loaded beam fixed at both ends is given in Table 5-3 as

$$R_m = \frac{16M_p}{L}$$

where the plastic moment

$$M_p = f_{dy} Z$$

Then the unit resistance (resistance per square inch of beam) is

$$r_m = \frac{16f_{dy}Z}{bL^2}$$

where

- Z = plastic section modulus, 1.932 in³
 f_{dy} = dynamic yield strength, 39,600 psi
 b = effective beam width, 1.44 inches
 L = length, 60 inches

Substituting in the above equation, it is found that

$$r_m = 236 \text{ psi}$$

The ratios t_1/T_N and t_2/T_N for the two triangles of the loading function meet the criteria for use of Eq. 5-55, pg. 5-58 to obtain the structural response, i.e.,

$$\left[\frac{C_1 P_r / r_m}{\frac{T_N}{\pi t_1} \sqrt{2\mu - 1}} \right]^2 + \frac{C_2 P_r / r_m}{\frac{T_N}{\pi t_2} \sqrt{2\mu - 1} + \frac{1 - \frac{1}{2\mu}}{1 + 0.7 \frac{T_N}{t_2}}} = 1$$

where

- r_m = maximum unit resistance of member, psi
 P_r = maximum pressure, psi
 t_1 = time of duration of first pulse, sec
 t_2 = time of duration of second pulse, sec
 μ = ductility ratio
 C_1 = ratio of peak pressure of first pulse to maximum pressure
 C_2 = ratio of peak pressure of second pulse to maximum pressure

A solution of Eq. 5-55 is obtained by trial and error. Successive values of μ are assumed until Eq. 5-55 is satisfied. A table may be set up as follows:

trial No.	μ	$(C_1 P_r / r_m / F_1)^2$	$C_2 P_r / r_m / F_2$	Sum of Terms
1	14	.239	.774	1.013
2	16	.208	.766	.968
3	15	.223	.769	.992 O.K.

The equivalent elastic deflection is given by

$$x_e = \frac{R_m}{K_e}$$

where, for the elastic-plastic range

$$K_e = K_E = \frac{307EI}{L^3} \text{ from Table 5-3}$$

$$R_m = \frac{16M_P}{L}$$

Then,

$$x_e = \frac{16M_P L^2}{307EI} = \frac{16(1.15)(1.68)(39,600)(60)^2}{307(29 \times 10^6)(2.52)}$$

$$= 0.1964 \text{ inch}$$

Maximum deflection is found from

$$x_m = \mu x_e = 0.1964\mu$$

and maximum strain is found from

$$\epsilon_m = \epsilon \mu_y = \mu \frac{f_y}{E} = 0.00137\mu$$

Substituting the calculated ductility ratio of 15, the maximum deflection and strain for a beam are 2.95 inches and 0.02 inch/inch, respectively.

5.6.2 Numerical Integration Technique for Determining Response of a Steel Beam Subjected to Blast Loading

a. Given

A Group 3 Shield S3x5.7 I-beam 60 inches long, fixed at both ends, and subjected to the blast loading determined in illustrative example 5.6.1. Although a multiple degree

of freedom analysis of the beams using the numerical integration technique would provide more detailed information, it would require a computer. The approximate single degree of freedom approach is used here.

b. Find

(1) The time at which the beam begins to yield, (2) the load acting on the beam at time of yielding, and (3) the maximum displacement of the beam by numerical integration.

c. Solution

The equation of motion for an elastic equivalent single degree of freedom system is given by Eq. 5-30, pg. 5-32.

$$K_{LM} M_t \ddot{X} + KX = F_t(t)$$

where

K_{LM} = the load-mass transformation factor

M_t = total mass of the real element, lb-sec²/in

K = elastic spring constant, lb/in

F_t = total load acting on element, lb

\ddot{X} = acceleration of the mass, in/sec²

X = displacement of the mass, inches

The total mass of the beam is

$$M_t = \frac{wL}{g} = \frac{0.475(60)}{386} = 0.073834 \text{ lb-sec}^2/\text{in}$$

The load mass factor, K_{LM} , is given in Table 5-3 as 0.78. The effective spring constant, K_E , is also given in Table 5-3.

$$K_E = \frac{307EI}{L^3} = \frac{307(29)(10)^6(2.52)}{60^3} = 103,868 \text{ lb/in}$$

Equation 5-60, pg. 5-62, is used in the numerical integration process,

$$\ddot{X} = \frac{F_t(t) - K_E X}{K_{LM} M_t} = \frac{F(t) - 103,868X}{0.05759}$$

The total force acting on the beam is the product of the blast pressure times the effective area over which the pressure acts, i.e.,

$$F_t(t) = P_t L b_e$$

where

P_t = blast pressure at time t , psi

L = length of beam, inches

b_e = effective width of beam, inches

At each time interval in the numerical integration process, the total force is calculated using the blast pressure loading from example 5.6.1, a beam length of 60 inches and an effective width of 1.44 inches. The numerical integration technique also requires the use of Eqs. 5-61 and 5-62, pg. 5-62.

$$\dot{X}_{n+1} = \dot{X}_n + \frac{\Delta t}{2} (\ddot{X}_n + \ddot{X}_{n+1})$$

$$X_{n+1} = X_n + \Delta t \dot{X}_n + \frac{(\Delta t)^2}{2} \ddot{X}_n + \beta (\ddot{X}_{n+1} - \ddot{X}_n) (\Delta t)^2$$

where the subscripts $n+1$ indicate values of X , \dot{X} and \ddot{X} at the time $t+\Delta t$. $\beta = 1/4$ is assumed for this example.

At $t = 0$, it is assumed that the beam has a velocity and displacement equal to zero. The applied force at $t = 0$ is

$$F_t = P_t L b_e = 3350(60)(1.44) = 289,440 \text{ lb}$$

The acceleration of the mass is given by

$$a = \frac{F_t - 103,868X}{0.05759} = 5,025,813 \text{ in/sec}^2$$

For the next step, the acceleration is assumed to be equal to that calculated from the initial step. Then from Eq. 5-62 with $X_n = \dot{X}_n = 0$, $\Delta t = 0.0001 \text{ sec}$ and $\ddot{X}_n = \ddot{X}_{n+1} = 5,025,813 \text{ in/sec}^2$,

$$\begin{aligned}
 x_{n+1} &= 0 + 0.0001(0) + \frac{(0.0001)^2}{2} (5,025,813) \\
 &= 0.025129
 \end{aligned}$$

and the calculated acceleration at $t = 0.0001$ is

$$\begin{aligned}
 \frac{F_t - 103,868x}{0.05759} &= \frac{173,664 - (103,868)(0.025129)}{0.05759} \\
 &= 2,970,166 \text{ in/sec}^2
 \end{aligned}$$

Obviously, the initial assumption for the acceleration was in error. The calculated value is used as the new assumed acceleration, and the calculations are repeated until the calculated value agrees with the assumed value within the desired degree of accuracy.

The velocity is calculated from Eq. 5-61 using the last cycle value of acceleration.

$$\dot{x}_{n+1} = \dot{x}_n + \frac{\Delta t}{2} (\ddot{x}_n + \ddot{x}_{n+1})$$

These calculations are summarized in a following table entitled Numerical Integration Summary. A similar process is repeated for each time increment with the acceleration calculated for the previous time step used as the initial assumed acceleration for the next.

The calculations are repeated until the displacement reaches the yield displacement

$$x_E = \frac{R_m}{K_E}$$

From Table 5-3, the maximum resistance is

$$\begin{aligned}
 R_m &= \frac{16M_P}{L} = \frac{16f_{dy}Z}{L} = \frac{16(39,600)(1.932)}{60} \\
 &= 20,401 \text{ lb}
 \end{aligned}$$

NUMERICAL INTEGRATION SUMMARY

Time (sec)	Assumed \ddot{x}_{n+1} (in/sec ²)	\dot{x} (in/sec)	x (in)	$F_t - K_{EX} \frac{K_{LM}^M}{t}$ (in/sec ²)
0	N.A.	0	0	5,025,813.77
0.0001	5,025,813.77 2,970,166.44 2,979,435.15 2,979,393.36		0.0251291 0.0199900 0.0200131 0.0200130	2,970,166.44 2,979,435.15 2,979,393.36 2,979,393.55
0.0002361	2,979,393.55 95,439.71 119,524.40 119,323.26 119,324.94		0.1020781 0.0887241 0.0888356 0.0888347 0.0888347	95,439.71 119,524.40 119,323.26 119,324.94 119,324.93
0.0004	119,324.93 - 64,919.79 - 62,688.01 - 62,715.04 - 62,714.71		0.1906034 0.1893659 0.1893809 0.1893807 0.1893807	- 64,919.79 - 62,688.01 - 62,715.04 - 62,714.71 - 62,714.72
0.0004115	- 62,714.72 - 75,527.56 - 75,526.79		0.1964578 0.1964574 0.1964574	- 75,527.56 - 75,526.79 - 75,526.79

and the effective spring constant is

$$K_E = \frac{307EI}{L^3} = 103,866 \text{ lb/in}$$

Therefore, the displacement at yield is

$$x_E = \frac{20,401}{103,866} = 0.1964 \text{ in}$$

The yield displacement is reached at $t = 0.0004115$ seconds.

The load acting on the beam at time of yielding is 16,056 lb.

In the plastic range, the acceleration is obtained by rearranging Eq. 5-31, pg. 5-32.

$$\ddot{x} = \frac{F(t) - R_m}{0.05759} = \frac{F(t) - 20,401}{0.05759}$$

The load during this time period is defined by

$$F(t) = \left(-\frac{187}{0.066} t + 187 \right) (1.44) (60) = -244,800t + 16,156$$

The maximum deflection, x_m , occurs when the velocity is zero. Therefore, from Eq. 5-61

$$0 = \dot{x}_n + \frac{\Delta t}{2} (\ddot{x}_n + \ddot{x}_{n+1})$$

Substituting, with $\Delta t = t - 0.0004115$,

$$0 = 614.9642631 + \frac{(t - 0.0004115)}{2}$$

$$\left[-75,526.79142 + \left(\frac{-244,800t + 16,156.8 - 20,401.92}{0.05759} \right) \right]$$

Solving,

$$t_m = 0.007243$$

$$\Delta t = 0.006832$$

$$\ddot{x} = -104,499.67$$

The maximum deflection is found using Eq. 5-62.

$$x_{n+1} = x_n + \Delta t \dot{x}_n + \frac{(\Delta t)^2}{2} \ddot{x}_n + \frac{1}{4} (\ddot{x}_{n+1} - \ddot{x}_n) (\Delta t)^2$$

Substituting,

$$\begin{aligned}
 X_{n+1} &= 0.1964574085 + (0.006832)(614.9642631) \\
 &\quad + \frac{(0.006832)^2}{2}(-75,526.79142) \\
 &\quad + \frac{1}{4}(-104,499.6704 + 75,526.79142)(0.006832)^2
 \end{aligned}$$

Solving,

$$X_{n+1} = 2.295 \text{ inches} = \text{maximum deflection}$$

Similar results could have been obtained by simply continuing the numerical integration process demonstrated in the summary table. For this example, the direct solution for t_m and X_m was more convenient.

5.6.3 Design of Roof Slab for Group 3 Type Suppressive Shield (Type I Construction Ref. 5-5)

a. Given

The same structure description and airblast loading as for illustrative example 5.6.1.

b. Find

Design a reinforced concrete roof slab for the shield using a ductility ratio of 6 (Ref Table 4-3).

c. Solution

Based on the airblast loading parameters of example 5.6.1,

$$i_r = 0.422 \text{ psi-sec}$$

$$C_1 = \frac{3350 - 187}{3350} = 0.944$$

$$C_2 = \frac{187}{3350} = 0.056$$

Some structural properties must be assumed or specified to start the design process. The designer can (1) assume trial section properties based on intuition or (2) use whatever aids are available to guide his choice of trial section

properties. The latter approach is chosen for this example. Since the quasi-static load is of fairly long duration, Eq. 5-37, pg. 5-53, might be used to obtain an initial estimate of the required R_m . The reflected impulse and the decay in pressure are neglected for this estimate.

$$r_m = P_{so} \left[\frac{2\mu}{2\mu-1} \right] = 187 \left[\frac{2(5)}{2(5)-1} \right] = 207.8 \text{ psi}$$

Select a somewhat lower resistance, e.g., 200 psi, for the first trial because of the anticipated larger influence in the decay in pressure. The following material properties are assumed from Table 4-1 and Table 4-2 for design purposes.

Concrete	$f'_c = 5000 \text{ psi}$
	$f'_{dc} = 6250 \text{ psi}$
Rebar	$f_y = 60,000 \text{ psi}$
	$f_{dy} = 72,000 \text{ psi}$
Structural Shapes	$f_y = 36,000 \text{ psi}$
	$f_{dy} = 39,600 \text{ psi}$

If the ends of the side wall beams are rigidly attached to the roof slab, they will provide some restraint of the outer edge of the slab. The moment capacity of the S3x5.7 beams is given by

$$M_{Ps} = \frac{f_{dy} Z}{\text{Beam Spacing}} = \frac{39,600(1.932)}{1.44} = 53,130 \text{ in-lb/in}$$

From Table 5-8, the maximum resistance of a circular slab with fixed edges is

$$R_m = 18.8(M_{PC} + M_{Ps})$$

Assuming the moment capacity of the slab at its edges is equal to the resistance provided by the beams, taking the radius of the slab to be 67.5 inches and $R_m = r_m A$, the required moment capacity at its center is

$$\begin{aligned}
 M_{PC} &= \frac{200\pi(a/2)^2}{18.8} - M_{PS} \\
 &= \frac{200(3.14)(67.5)^2}{18.8} - 53,130 = 99,145 \text{ in-lb/in}
 \end{aligned}$$

In order to determine the required depth of the slab, a reinforcing steel ratio must be assumed. Try

$$p = A_s/bd = 0.01$$

Equation 5-7, pg. 5-13, gives the moment capacity of a reinforced concrete member as

$$M_P = pf_{dy}bd^2 \left[1 - 0.59p \frac{f_{dy}}{f'_{dc}} \right]$$

This equation may be solved directly for "d" but by manipulating, Equation 5-7 as shown above the following simple substitutions allow an easier solution.

$$m = \frac{f_{dy}}{0.85f'_{dc}} = \frac{66000}{(0.85)(6250)} = 12.423$$

$$\begin{aligned}
 K_u &= pf_{dy} \left[1 - \frac{pm}{2} \right] = (0.01)(66000) \left[1 - \frac{(0.01)(12.423)}{2} \right] \\
 &= 619.004
 \end{aligned}$$

$$bd^2 = \frac{M_{PC}}{K_u} \quad \text{where } b = 1"$$

$$d = \sqrt{\frac{99145}{619.004}} = 12.65"$$

Normally, the 12.65 inches would be rounded off to some practical depth like 13.0 inches; however, it will be retained for our first trial section.

From Eq. 5-8, pg. 5-14, the moment of inertia of the slab is

$$\begin{aligned}
 I_a &= \frac{bd^3}{2} [5.5p + 0.083] \\
 &= \frac{1(12.65)^3}{2} [5.5 \times 0.01 + 0.083] = 139.68 \text{ in}^4/\text{in}
 \end{aligned}$$

From Eq. 4-1, pg. 4-6, the modulus of elasticity for 150 lb/ft³ concrete is

$$E_c = 33w^{1.5} \sqrt{f'_c}$$

$$= 33(150)^{1.5} (5000)^{0.5} = 4.29 \times 10^6 \text{ psi}$$

From Table 5-8, the stiffness of the concrete slab is

$$K = \frac{216EI}{a^2} = \frac{216(4.29 \times 10^6)(139.68)}{(135)^2} = 7,101,952 \text{ lb/in}$$

$$K_{LM} = 0.65 \text{ (for elastic-plastic range)}$$

Assuming an overall slab thickness of 16", the total mass of the slab is

$$M_t = \frac{16(3.14)(67.5)^2(150)}{1728(386)} = 51.50 \text{ lb-sec}^2/\text{in}$$

From Eq. 5-33, pg. 5-32, the period of vibration of the slab is

$$T_N = 2\pi \left[\frac{K_{LM} M_t}{K} \right]^{1/2}$$

$$= 6.28 \left[\frac{0.65(51.50)}{7,101,952} \right]^{1/2} = 0.0136 \text{ sec}$$

The next step is to determine the response of the slab to the blast pressure loading. The ratios t_o/T_N for the two triangular components of the loading function indicate that Eq. 5-55, pg. 5-58, is the appropriate response equation to use.

$$\left[\frac{\frac{C_1 P_r}{r_m}}{\frac{T_N}{\pi t_1} \sqrt{2\mu-1}} \right]^2 + \frac{\frac{C_2 P_r}{r_m}}{\frac{T_N}{\pi t_2} \sqrt{2\mu-1} + \frac{1 - \frac{1}{2\mu}}{1 + \frac{0.7T_N}{t_2}}} = 1$$

Substituting,

$$\left[\frac{\frac{0.944(3350)}{200}}{\frac{0.0136}{3.14(0.00025)} \sqrt{2(6)-1}} \right]^2 + \frac{\frac{0.056(3350)}{200}}{\frac{0.0136}{3.14(0.066)} \sqrt{2(6)-1} + \frac{1 - \frac{1}{2(6)}}{1 + \frac{0.7(0.0136)}{0.066}}} = 1$$

$$= 0.076 + 0.921 = 0.997$$

Additional trials are not necessary in this case. A value of $r_m = 200$ psi results in a required moment capacity of 99,145 in-lb/in, $d = 12.65$ inches, $I = 139.68$ in⁴/in, $K = 7,101,954$ lb/in, $T_N = 0.0136$ sec and Eq. 5-55 yields a sum of 0.997.

The shear capacity of the slab depends on the radial tension loads applied by the wall beams. If properly anchored and/or attached to the wall beams, it might be assumed that the circular col. base plates and reinforcing steel in the slab resist all radial forces and prevent tension cracks in the concrete. On the basis of this assumption, the full effective depth of the slab is available to resist the maximum shear force. From Table 5-8, the dynamic reaction at the edge of the slab is

$$V = 0.36F + 0.64R_m$$

where

$$R_m = 200(3.14)(67.5)^2 = 2,861,325 \text{ lbs}$$

For purposes of analysis, it is assumed that the load at the time of maximum response is equal to the quasi-static pressure. Then

$$F = 187(3.14)(67.5)^2 = 2,676,696 \text{ lbs}$$

Substituting in the above equation,

$$V = 0.36(2,676,696) + 0.64(2,861,325) = 2,794,858 \text{ lbs}$$

The reaction per inch of support is

$$\frac{2,794,858}{2(3.14)(67.5)} = 6593 \text{ lbs/in}$$

Using the criteria of Ref. 5-7 determine the required depth, d , for diagonal tension assuming ductile mode. The maximum allowable shear stress with shear reinforcing is

$$v_u = 11.5\sqrt{f'_c} = 11.5\sqrt{5000} = 813 \text{ psi}$$

Solving for d ,

$$d = \frac{6593}{(1)(813)} = 8.10 < 12.65 \text{ inches}$$

Next check the required depth of the roof slab for a possible direct shear failure as determined by Eq. 5-9, pg. 5-14

$$v_d = \frac{V_d}{bd} = 0.18f'_c$$

As direct shear is a brittle mode of failure a ductility ratio of only 1.3 (ref. Table 4-3, pg. 4-15) is allowed. Table 4-2 recommends a 10% increase in the direct shear strength of members due to rapid loading.

Substituting in Eq. 5-55, as noted on pg. 5-80, the value of μ of 1.3 and solving for a new r_m we see that $r_m = 300$ psi satisfies the equation

Determining our new shear reaction we have

$$V = 0.36F + 0.64R_m$$

where

$$R_m = 300 (3.14) (67.5)^2 = 4,291,987\#$$

and

$$F = 187 (3.14) (67.5)^2 = 2,861,325\#$$

therefore

$$V = (0.36)(2,676,696) + (0.64)(4,291,987) = 3,710,482\#$$

The reaction per inch of support is

$$\frac{3,710,482}{2(3.14)(67.5)} = 8753\#/\text{in}$$

Solving for d from Eq. 5-9 and allowing for the 10% increase in f'_c

$$d = \frac{V_d}{(1.1)(.18)(f'_c)} = \frac{8753}{(1.1)(.18)(5000)} = 8.84 < 12.65 \text{ inches}$$

Therefore bending controls the effective depth of the roof slab. For an effective depth of 12.65 inches and $M_{PC} = 99,145$ in-lb/in, the required steel ratio in the center of the roof (top steel) is also found from Eq. 5-7.

$$M_{PC} = pf_{dy}bd^2 \left[1 - 0.59p \frac{f_{dy}}{f'_{dc}} \right]$$

This equation may be solved directly for "p", but the following simple substitutions allow an easier solution.

$$\text{Let } m = \frac{f_{dy}}{0.85 f'_{dc}} = \frac{72000}{(0.85)(6250)} = 13.553$$

$$K_u = \frac{M_{Pc}}{bd^2} = \frac{99145}{(1)(12.65)^2} = 619.569$$

$$p = \frac{1}{m} \left[1 - \sqrt{1 - \frac{2mK_u}{f_{dy}}} \right]$$

$$= \frac{1}{13.553} \left[1 - \sqrt{1 - \frac{(2)(13.533)(619.569)}{72000}} \right] = .0092$$

$$A_s = (0.0092)(1)(12.65) = .116 \text{ in}^2/\text{in} = 1.392 \text{ in}^2/\text{ft.}$$

Use two layers #5 @ 5" c.c. ($A_s = 1.44 \text{ in}^2/\text{ft.}$)

$$\text{Actual } p = 1.44/(12)(1)(12.65) = 0.0095$$

The wall beams provide a resistance of 53,130 in-lb/in and the moment capacity of the slab at the supports must be at least equal to that. Assume $M_{ps} = 55,000 \text{ in-lb/in}$. For an effective depth of 12.65 inches, the required steel ratio at the support (bottom steel) is found from Eq. 5-7, pg. 5-13.

$$55,000 = p(72000)(1)(12.65)^2 \left[1 - 0.59p \frac{72,000}{6,250} \right]$$

or

$$p = 0.0049 \text{ (say } 0.0095 \text{ to provide additional restraint for radial reaction of wall beams.) See Example 5.6.6}$$

$$A_s = 0.0095(1)(12.65) = \underline{\underline{0.120 \text{ in}^2/\text{in}}}$$

The allowable shear stress in the concrete is given by Eq. 5-10, pg. 5-15.

$$v_c = (1.9\sqrt{f'_c} + 2500pdv_c/M_c)$$

where dv_c/M_c must be less than 1.0, and v_c less than $3.5\sqrt{f'_c}$ psi. Substituting previously calculated values,

$$dv_c/M_c = 12.65(6593)/55,000 = 1.52 > 1. \text{ Use } 1.0.$$

Then,

$$v_c = \left[1.9\sqrt{5000} + 2500(0.0095)(1) \right] = 158 \text{ psi} < 3.5\sqrt{f'_c} = 248 \text{ psi} \quad \underline{\text{OK}}$$

Therefore, the shear capacity of the concrete is

$$V_c = v_c b d = 158(1)(12.65) = 1999 \text{ lb/in}$$

The required shear reinforcing capacity is

$$V_s = V_u - V_c = 6593 - 1999 = 4594 \text{ lb/in}$$

The shear stress to be resisted by the shear reinforcing is

$$v_s = \frac{4594}{(1)(12.65)} = 363 \text{ psi} < 8\sqrt{5000} = 566 \text{ psi OK}$$

The size and spacing requirements are found from Eq. 5-11, pg. 5-15.

$$V_s = \frac{d A_v f_{dy}}{s}$$

and

$$\frac{A_v}{s} = \frac{4594}{12.6(72000)} = 0.0050 \text{ in}^2/\text{in for a 1 inch slab width}$$

In summary, the tensile reinforcing steel should be $0.120 \text{ in}^2/\text{in}$ at the center of the slab and at the supports. Vertical shear reinforcing should provide $0.0050 \text{ in}^2/\text{in}^2$ of slab surface area near the support. The overall depth of the slab must be sufficient to provide proper protection for the reinforcing steel and will depend upon the size and number of layers of reinforcing. This preliminary design could be made more conservative by neglecting the shear strength of the concrete in computing shear reinforcing requirements. It is possible that radial loads applied to the slab could cause cracking of the concrete and loss of shear strength, so care should be taken in detailing the reinforcing steel to assure full development of all bars through adequate embedment or mechanical anchorage. See Example 5.6.6 for reinforcing requirements due to radial loads applied by wall beams.

5.6.4 Analysis of Shield Group 6A Designa. Given

Shield Group 6A is a sphere with an inside diameter of 2 feet and a wall thickness of 1/4 inch. The shield is designed for a 13.63 ounce charge of Pentolite. It is made from mild steel with the following properties.

E = modulus of elasticity, 29×10^6 psi

ν = Poisson's ratio, $1/3$

f_{dy} = dynamic yield strength, 39,600 psi

ρ = mass density, 7.36×10^{-4} lb-sec²/in⁴

b. Find

The strain and deformation due to detonation of the design charge weight.

c. Solution

The equivalent TNT charge weight for 13.63 ounces of Pentolite is obtained from Table 3-1 and Eq. 3-1, pg. 3-4.

$$W = \frac{13.63(1.129)}{16} = 0.962 \text{ lb TNT}$$

and

$$W^{1/3} = 0.987 \text{ lb}^{1/3}$$

Assuming the charge is at the center of the shield, the scaled distance to the wall is

$$Z = R/W^{1/3} = 1/0.987 = 1.013 \text{ ft/lb}^{1/3}$$

From Fig. 3-6, the peak reflected pressure, P_r , is 7000 psi and the scaled impulse is

$$i_r/W^{1/3} = 0.2 \text{ psi-sec/lb}^{1/3}$$

Then

$$i_r = 0.197 \text{ psi-sec}$$

and

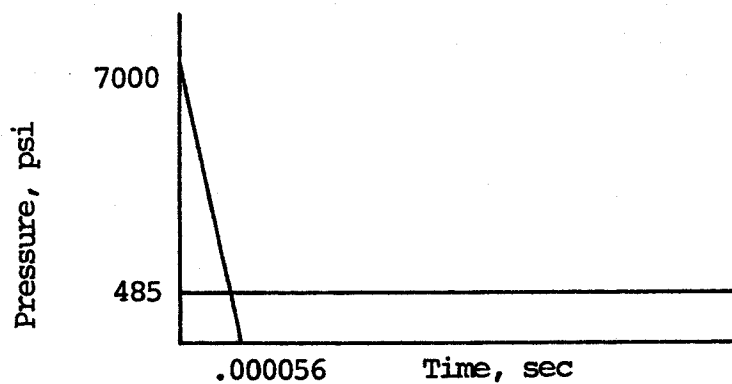
$$t_1 = 2i_r/P_r = (2)0.197/7000 = 0.000056 \text{ sec}$$

From Fig. 3-9, for

$$\frac{W}{V} = \frac{0.962}{4.188} = 0.230 \text{ lb/ft}^3$$

$$P_{qs} = 485 \text{ psi}$$

The blast pressure loading on the spherical chamber is as shown below. As indicated in the loading diagram, little or no venting would occur in this shield.



Blast Loading of a Group 6A Shield

$$C_1 = \frac{7000 - 485}{7000} = .931$$

$$C_2 = \frac{485}{7000} = .069$$

The structure responds dynamically to both the reflected pressure pulse and the quasi-static pressure. Assume the spherical chamber responds only in its fundamental mode of vibration, a simultaneous radial motion of all points on its surface. The natural period of the sphere is then obtained from Eq. 5-24, pg. 5-26, i.e.,

$$T_n = 2\pi \sqrt{\frac{\rho a^2 (1-2)}{2E}}$$

where

a = radius of sphere, 12 inches

The natural period of the sphere is

$$T_n = 2\pi \left[\frac{(7.36 \times 10^{-4}) (12)^2 (.667)}{(2) (29 \times 10^6)} \right]^{1/2}$$

$$= .000219 \text{ sec}$$

Assuming thin shell response, the unit resistance of the sphere is determined from Eq. 5-17, pg. 5-20, i.e.

$$f_{dy} = \frac{r_m a}{2 t}$$

where

r_m = unit resistance, psi

t = wall thickness, .25 inches

Therefore

$$r_m = \frac{(2) (39600) (.25)}{12} = 1650 \text{ psi}$$

The next step is to determine the response of the sphere to blast pressure loading. The loading diagram indicates that Eq. 5-54, pg. 5-57, is the appropriate equation to use.

$$\left[\frac{C_1 P_r/r_m}{\frac{T_N}{\pi t_1} \sqrt{2\mu - 1}} \right]^2 + \frac{C_2 P_r/r_m}{1 - \frac{1}{2\mu}} = 1$$

The use of the charts of Appendix B simplifies the solution to this equation. From Fig. B-34, pg. B-40, for $C_1 = 0.93$ and $C_2 = 0.07$ with

$$P_r/r_m = 7000/1650 = 4.24$$

$$t_1/T_N = .000056/.000219 = .256$$

the ductility ratio, μ , is determined to be approximately 7.8.

The radial deflection, X_e , of the spherical chamber at the membrane yield stress is from Ref. 5-18.

$$\begin{aligned} X_e &= (f_{dy}) (a) (1-\nu) / E \\ &= (39600) (12) (.667) / (29 \times 10^6) = .011 \text{ inches} \end{aligned}$$

The maximum deflection is found from

$$X_m = \mu X_e = (7.8) (.011) = .086 \text{ inches}$$

5.6.5 Response of Removable Column in 81-mm Suppressive Shield

a. Given

The Milan 81-mm suppressive shield is a steel frame and panel structure with inside dimensions of 14 feet by 14 feet by 12.4 feet. All vertical frame members (except corners) are 8 x 6 x 1/4 structural steel tubing. Horizontal ceiling members are 8 x 6 x 3/8 structural steel tubing. Panels are mounted from the inside and restrained against the frame. One of the vertical frame members is removable to provide a larger access opening into the shield. Pages A-86 thru A-118 provide details of this shield.

A charge equivalent to 5.25 pounds of TNT is assumed to be located at the center of the shield. The effective vent ratio for the shield is $\alpha_{\text{eff}} = 0.043$. The volume of the structure is 2430.4 ft^3 and the vented surface area, A_i , 890.4 ft^2 . Atmospheric pressure, P_o , is assumed to be 14.7 psi, and the sound velocity in air, a_o , is 1117 ft/sec.

b. Find

The maximum axial tension in the removable column and its bending response to the blast loading.

c. Solution

The first step is to compute blast loads for both the roof and walls. For the roof, the scaled distance is

$$Z = R/W^{1/3} = 6.20/5.25^{1/3} = 3.57 \text{ ft/lb}^{1/3}$$

From Fig. 3-6, the peak reflected pressure, P_r , is 230 psi. Scaled reflected impulse, $i_r/W^{1/3}$, is 0.045 psi-sec/lb^{1/3}. Therefore, the impulse is

$$i_r = 0.045(1.738) = 0.078 \text{ psi-sec}$$

The duration of the reflected pulse is obtained from Eq. 3-4, pg. 3-14.

$$t_r = 2i_r/P_r = 2(0.078)/230 = 0.00068 \text{ sec}$$

For the walls, the scaled distance is

$$Z = R/W^{1/3} = 7/5.25^{1/3} = 4.03 \text{ ft/lb}^{1/3}$$

and from Fig. 3-6

$$P_r = 175 \text{ psi}$$

$$i_r/W^{1/3} = 0.038 \text{ psi-sec/lb}^{1/3}$$

$$i_r = 0.066 \text{ psi-sec}$$

$$t_r = 2i_r/P_r = 0.00075 \text{ sec}$$

For

$$\frac{W}{V} = \frac{5.25}{(14)(14)(12.4)} = 0.0022 \text{ lb/cf}$$

Figure 3-9 indicates the quasi-static pressure to be

$$P_{qs} = 26 \text{ psi}$$

Next, substitute

$$P_{qs} = 26 \text{ psi}$$

$$P_o = 14.7 \text{ psi}$$

into the equation for scaled maximum pressure, pg. 3-23

$$\bar{P} = (P_{qs} + P_o)/P_o = (26 + 14.7)/14.7 = 2.77$$

Using 2.77 as the ordinate to the plot of Fig. 3-10, it is found that

$$t_b a_o \alpha_e A_i / V = 0.48$$

Substituting

$$\alpha_{eff} = 0.043$$

$$V = 2430.4 \text{ ft}^3$$

$$A_i = 890.4 \text{ ft}^2$$

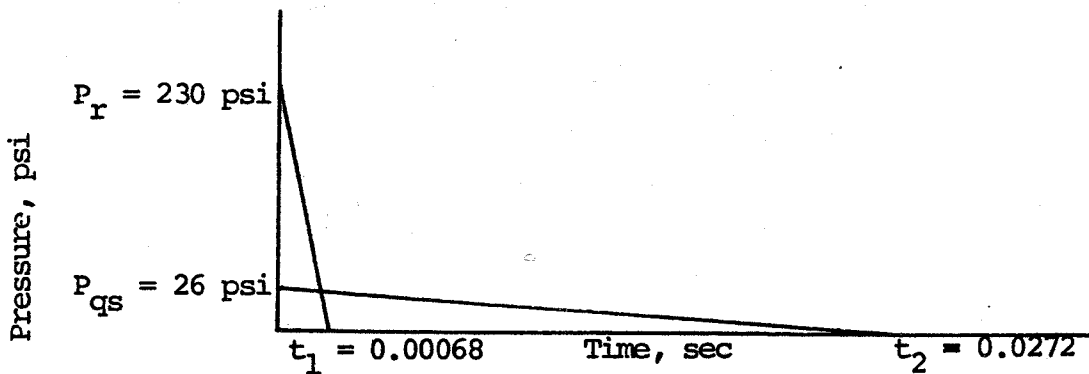
$$a_o = 1117 \text{ ft/sec}$$

and solving for the blowdown time, it is found that

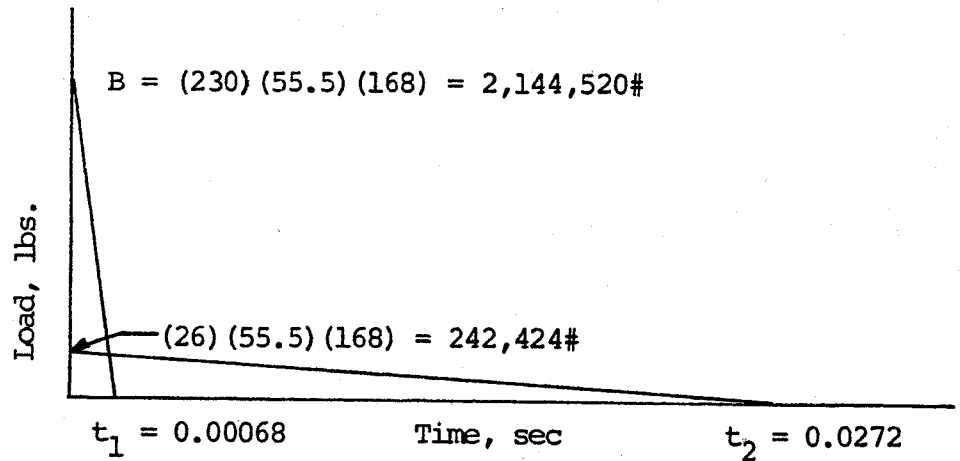
$$t_b = 0.0272 \text{ sec}$$

The shield panels are the primary means of transferring the blast pressure loads into the roof beams and side columns. As a conservative check of the loads applied to the removable column, assume that the panels are infinitely stiff.

The roof beam is 168 inches long and has a plastic section modulus, Z , equal to 27.02 inches³. The beam reacts the blast loads from two panels with a tributary width of 55.5 inches. Based on this data, the airblast loads acting on the roof beam are shown in the following sketches



Unit Airblast Loads Acting on the Roof Beam



Total Airblast Loads Acting on the Roof Beam

The maximum tension in the removable column is equal to the maximum dynamic reaction of the roof beam. The dynamic reaction for the uniformly loaded fixed end beam is given in Table 5-3, pg. 5-34,

$$V_{\max} = 0.38 R_m + 0.12F$$

The maximum bending resistance from Table 5-3 for the roof beam fixed at both ends is

$$R_m = \frac{16 M_p}{L} = \frac{16Z f_{dy}}{L} = \frac{(16) (27.02) (39,600)}{168} = 101,904 \text{ lb}$$

The natural frequency of the roof beam is given in Fig. 5-8 as

$$\omega_N = 22.4 \frac{E I}{m L^4}$$

where

E = modulus of elasticity, 29×10^6 psi

I = moment of inertia, 79.7 in.^4

m = mass per linear inch of beam, $\text{lb-sec}^2/\text{in}^2$

L = length of beam, 168 inches

Since the panel weight is distributed along the beam, it must also be included. The weight of a panel is 32.6 lb/ft^2 , and its width is 49.5 inches. The $8" \times 6" \times 3/8"$ beam weighs 2.61 lb/in. The mass per inch is then

$$m = \left[2.61 + \frac{(49.5)(32.6)}{144} \right] / 386 = 0.0358 \text{ lb-sec}^2/\text{in}^2$$

and

$$\omega_N = 22.4 \left[\frac{(29 \times 10^6)(79.7)}{(0.0358)(168^4)} \right]^{1/2} = 201.658 \text{ rad/sec}$$

The natural period is

$$T_N = \frac{2\pi}{\omega_N} = 0.03116 \text{ sec}$$

The maximum tension in the removable column is equal to the maximum dynamic reaction of the roof beam. From Table 5-2, pg. 5-34,

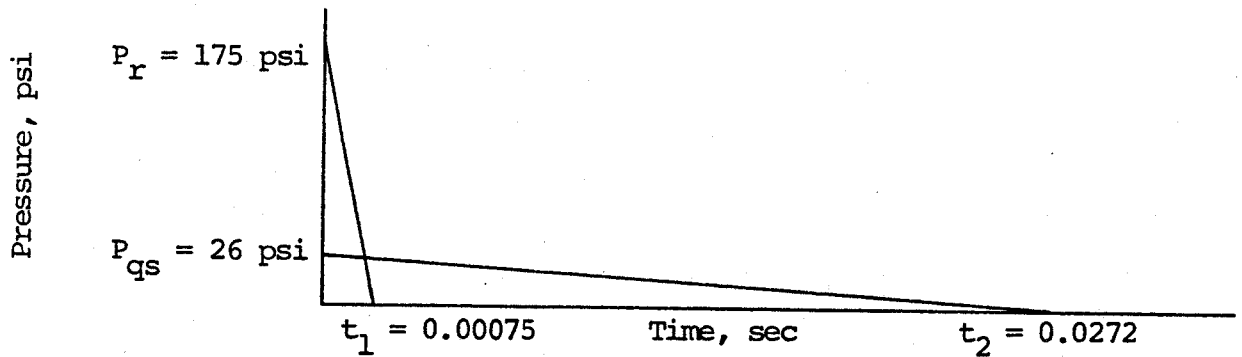
$$V_{\max} = 0.38 R_m + 0.12F$$

The total force, F , is time-dependent and its value at the time the roof beam yields should be used in the above equation; however, for a conservative estimate, the total quasi-static load is used. Since the natural period of the roof beam is very much longer than the duration of the peak reflected pressure, the peak reflected pressure is neglected.

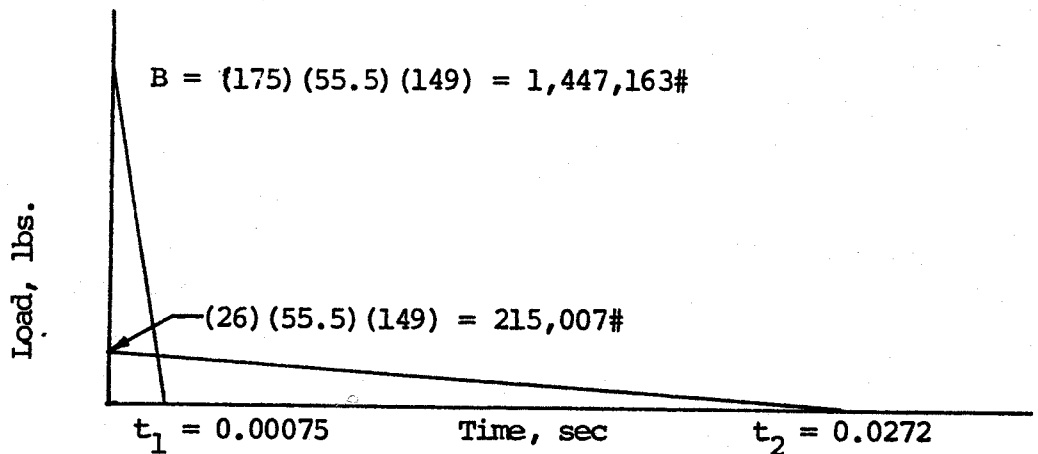
Therefore

$$V_{\max} = (0.38)(101,904) + (0.12)(215007) = 64,524 \text{ lg}$$

The horizontal load on the removable column is applied through the panels in the same fashion as described above for the roof beams. The removable column is 149 inches long and has a plastic section modulus, Z , equal to 18.66 in^3 . Based on this data, the blast loading on the removable column is shown in the following sketches.



Unit Airblast Loads Acting on the Removable Column



Total Airblast Loads Acting on the Removable Column

From the sketch,

$$C_1 = (1,447,163 - 215,007)/1,447,163 = .852$$

$$C_2 = 215,007/1,447,163 = .148$$

The column is assumed to be simply-supported at the base and fixed at the roof. From Table 5-3, the maximum bending resistance of the column is

$$R_m = \frac{12 M_P}{L} = \frac{12 Z f_{dy}}{L} = \frac{(12)(18.66)(39,600)}{149} = 59,512 \text{ lb}$$

The natural frequency of the element is given in Fig. 5-8 as

$$\omega_N = 15.4 \sqrt{\frac{EI}{mL^4}}$$

where

E = modulus of elasticity, 29×10^6 psi

I = moment of inertia, 58.4 in^4

m = mass per linear inch of beam, $\text{lb-sec}^2/\text{in}^2$

L = length of beam, 149 inches

As with the roof beam, the panel weight is distributed along the beam. The weight of a panel is 32.6 lb/ft^2 , and its width is 49.5 inches. The $8" \times 6" \times 1/4"$ column weighs 1.84 lb/in . The mass per inch is then

$$m = \left[1.84 + \frac{49.5 (32.6)}{144} \right] / 386 = 0.0338 \text{ lb-sec}^2/\text{in}^2$$

and

$$\omega_N = 155.276 \text{ rad/sec}$$

The natural period is

$$T_N = \frac{2}{\omega_N} = 0.04046 \text{ sec}$$

The response of an elastic-plastic system to short duration and quasi-static triangular pulses is given by Eq. 5-55, pg. 5-58.

$$\left[\frac{C_1 B/R_m}{\frac{T_N}{\pi t_1} \sqrt{2\mu - 1}} \right]^2 + \left[\frac{C_2 B/R_m}{\frac{T_N}{\pi t_2} \sqrt{2\mu - 1} + \frac{1 - \frac{1}{2\mu}}{1 + 0.7 \frac{T_N}{t_2}}} \right] = 1$$

where

R_m = maximum resistance of member, lb

B = maximum load on member, lb

T_N = natural period of element, sec

t_1 = time of duration of short pulse, sec

t_2 = time of duration of quasi-static pulse, sec

μ = ductility ratio

C_1 = ratio of the peak short duration force to the peak total force

C_2 = ratio of peak quasi-static force to the peak total force

A solution of Eq. 5-55 is obtained by trial and error. Successive values of μ are assumed until the equation is satisfied. Only the final calculation for $\mu = 23.8$ is shown below.

$$\begin{aligned}
 & \left[\frac{\frac{0.852(1,447,163)}{59,512}}{\frac{0.04046}{3.14(0.00075)} \sqrt{2(23.8)-1}} \right]^2 \\
 & + \left[\frac{\frac{(.148)(1,447,163)}{59,512}}{\frac{0.04046}{3.14(0.0272)} \sqrt{2(23.8)-1} + \frac{1 - \frac{2(23.8)}{1}}{1 + 0.7 \frac{(0.04046)}{(0.0272)}}} \right] \\
 & = 1.005
 \end{aligned}$$

The equivalent elastic displacement is given by

$$x_E = \frac{R_m}{K_E}$$

where for the elastic-plastic range

$$K_E = \frac{160 EI}{L^3} \quad \text{from Table 5-3}$$

or

$$K_E = \frac{160 (29 \times 10^6) (58.4)}{(149)^3} = 81,917 \text{ lb/in}$$

then

$$x_E = \frac{59,512}{81,917} = 0.726 \text{ inches}$$

and the maximum displacement is

$$X_m = \mu X_E = 23.8 (0.726) = 17.3"$$

Based on the assumption of infinitely stiff panels, the maximum displacement is conservative. A less conservative approach would involve determining the dynamic reactions of the panels as loads on the roof beams and columns.

5.6.6 Analysis of Base Plate Ring and Reinforcing Steel in Foundation Slab for Group 3 Suppressive Shield

a. Given

The same structure description and pressure loading as for illustrative example 5.6.1. The base plate ring has an outside diameter of 149 inches and an inside diameter of 128 inches. It is 1 inch thick. Pages A-12 thru A-27 provide details of the Group 3 Type Shield.

b. Find

The required reinforcing steel to resist the wall beam reactions.

c. Solution

Assume that the annular base plate and top rebars resist the entire radial load applied by the wall beams. From Table 5-3, the dynamic reaction at the ends of the interlocking I-beams is

$$V = 0.38R_m + 0.12F$$

where

$$R_m = \frac{16M_P}{L} = \frac{16Zf_{dy}}{L} = \frac{16(1.932)(39,600)}{60} = 20,401 \text{ lb/beam}$$

Since the natural period of the I-beam is much longer than the duration of the peak reflected pressure, only the quasi-static pressure load is considered in determining the load. Then

$$F = 187(1.44)(60) = 16,156 \text{ lb/beam}$$

Substituting in the above equation for the maximum reaction,

$$V = 0.38(20,401) + 0.12(16,156) = 9,692 \text{ lb/beam}$$

There are 296 beams at an effective diameter of 135 inches.

The equivalent uniform radial load is

$$P = \frac{(\text{load/beam})(\text{number of beams})}{\text{circumference of ring}} = \frac{9692(296)}{135\pi} = 6764 \text{ lb/in}$$

The elastic deflection of the outer radius of the ring under a uniform radial pressure is (Ref. 5-18)

$$u_r = \frac{qR_o}{E} \left[\frac{2 R_i^2}{R_o^2 - R_i^2} \right]$$

where

R_o = outer radius, 74.5 inches

R_i = inner radius, 64 inches

E = modulus of elasticity, 29×10^6 psi

q = uniform radial pressure, psi

The pressure q is multiplied by the 1 inch height, h , of the ring to obtain the radial load per inch of circumference of the ring, i.e.,

$$qh = F_R = \text{radial load per inch of ring}$$

or

$$q = \frac{F_R}{h} = \frac{F_R}{1.0}$$

Substituting

$$u_r = \frac{F_R(74.5)}{1.00(29 \times 10^6)} \left[\frac{2(64)^2}{(74.5)^2 - (64)^2} \right]$$

$$u_r = 14.471 \times 10^{-6} F_R$$

The elastic deflection of the reinforcement is

$$u_b = \frac{F_B L}{A_S E} = \frac{F_B R_O}{A_S E} = \frac{(74.5) F_B}{29(10)^6 A_S} = 2.569(10)^{-6} \frac{F_B}{A_S}$$

where

F_B = radial load carried by the reinforcement per inch of circumference assuming no radial deformation due to bending

A_S = cross sectional area of reinforcement available to resist tensile loads per inch of circumference

The area of reinforcement available to resist tensile forces is determined from example 5.6.3, pg. 5-82.

$$A_S = (.0095 - .0049)(1)(12.65) = 0.058 \text{ in}^2/\text{in}$$

Therefore

$$u_b = \frac{2.569(10)^{-6}}{0.058} F_B = 44.293(10)^{-6} F_B$$

Since the base plate and top layer of reinforcement are assumed to act together,

$$u_b = u_r$$

$$44.293(10)^{-6} F_B = 19.295(10)^{-6} F_R$$

$$F_R = 2.30 F_B$$

The above ratio of loads per inch of circumference in the ring and reinforcement is only good in the elastic range. The dynamic yield stresses for the ring and reinforcement are given in Table 4-1, pg. 4-5.

$$f_{dyR} = 39600 \text{ psi for the ring}$$

$$f_{dyB} = 72000 \text{ psi for the reinforcement}$$

The first step is to determine whether the combination of ring and reinforcement can resist the radial tension loads in the elastic range,

$$F_B + F_R = P$$

$$F_B + 2.30 F_B = 6764 \text{ lb/in}$$

$$F_B = 2050 \text{ lb/in in the reinf.}$$

then

$$F_R = 4714 \text{ lb/in in the ring}$$

The stress in the reinforcement is

$$\sigma_B = \frac{F_B}{A_S} = 35345 \text{ psi} < 72000 \text{ psi}$$

The maximum stress in the steel ring is given by Case 1a, page 504, Ref. 5-18.

$$\begin{aligned} \sigma_R &= \frac{F_R}{r} \left[\frac{2R_o^2}{R_o^2 - R_i^2} \right] \\ &= \frac{4714}{1.0} \left[\frac{2(74.5)^2}{(74.5)^2 - (64)^2} \right] = 35,983 \text{ psi} < 39,600 \text{ psi} \end{aligned}$$

The stresses for both the ring and reinforcement are below the yield point, so the combined system is adequate to resist the radial beam reactions.

An economical redesign would allow the ring, with its lower dynamic yield stress, to respond in the plastic range. This would allow a reduction in the reinforcing steel.

In general the reaction of the ring and reinforcing steel to the radial tensile loads could be developed in three stages.

1. The elastic range response for both the ring and reinforcement.
2. The combination of plastic range response for the ring ($f_{dyR} = 39600$ psi) and the elastic range for the reinforcing steel ($\nabla_{max} < f_{dyE} = 72000$ psi).
3. The plastic range response for both the ring and reinforcement.

5.6.7 Design of Upper Connection for Removable Column in 81-mm Suppressive Shield

a. Given

The Milan 81-mm suppressive shield is a steel frame and panel structure with inside dimensions of 14 feet by 14 feet by 12.4 feet. The side columns in the shield area are 8 x 6 x 1/4 structural tubes. A description of the shield and its design parameters are given in Example 5.6.5.

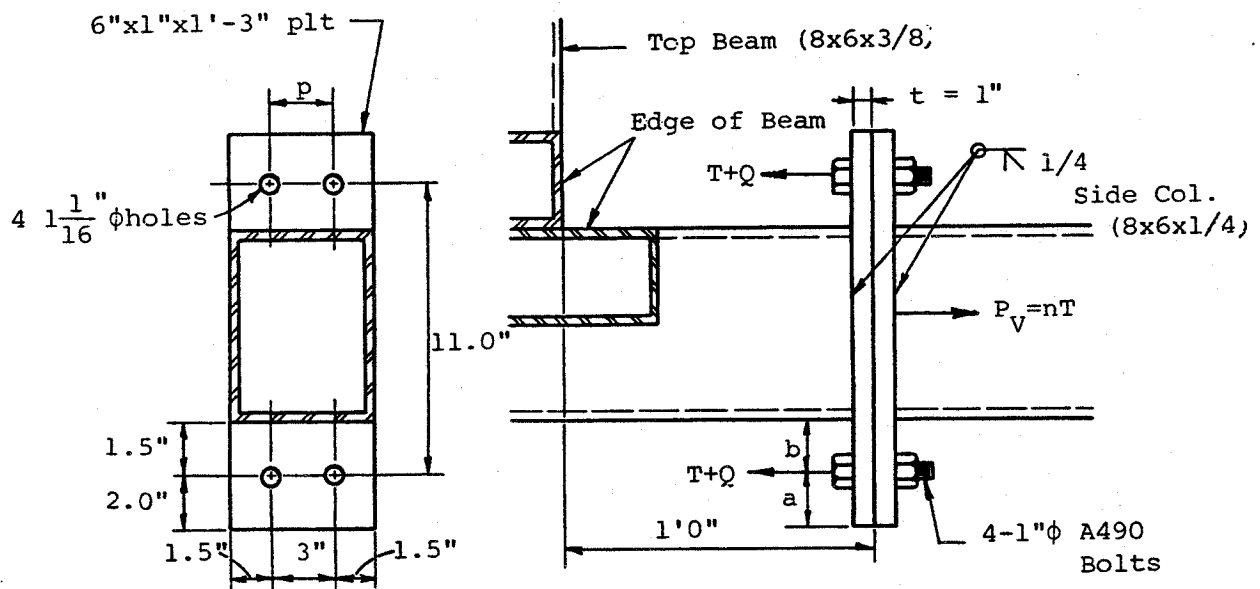
b. Find

Using the loads derived in Example 5.6.5, analyze the upper connection of the removable column. Assume the upper end of the column is fixed and the lower end is simply supported. The upper connection is located 1 foot below the roof beam.

c. Solution

Without a finite element dynamic analysis of the shield, it is difficult to predict the exact phasing of the various axial and bending components of loads applied to the connection. For this reason, the analysis presented here evaluates the connections ability to develop separately the columns full plastic moment, full axial load capacity, and the combination of bending plus axial load from Example 5.6.5.

Details of the upper connection are shown below.



Removable Column Upper Connection Details

The axial load capacity of the column is

$$P_e = f_{dy} A = 6.48(39,600) = 256,608 \text{ lbs}$$

Approximate load per bolt without prying action is

$$P_b = \frac{256,608}{4} = 64,152 \text{ lbs}$$

The fully plastic moment capacity of the 8 x 6 x 1/4 column is

$$M_p = f_{dy} Z = 39,600(18.66) = 738,936 \text{ in-lb}$$

The load per bolt without prying action for an 11.0-inch lever arm is

$$P_b = \frac{738,936}{2(11)} = 33,588 \text{ lbs}$$

Bolt tension increase, Q , due to prying action is found from Eq. 13.6.37 of Ref. 5-3.

$$\frac{Q}{P_b} = \frac{\frac{1}{2} - \frac{pt^4}{30ab^2A_b}}{\frac{a}{b}\left(\frac{a}{3b} + 1\right) + \frac{pt^4}{6ab^2A_b}}$$

where, referring to the sketch of the connection,

$$p = 3 \text{ inches}$$

$$a = 2 \text{ inches}$$

$$b = 1.5 \text{ inches}$$

$$A_b = \text{area of bolt} = 0.785 \text{ in}^2$$

$$t = 1 \text{ inch}$$

Then

$$\frac{Q}{P_b} = \frac{\frac{1}{2} - \frac{3(1.0)^4}{30(2)(1.5)^2(0.785)}}{\frac{2}{1.5}\left(\frac{2}{3 \times 1.5} + 1\right) + \frac{3(1.0)^4}{6(2)(1.5)^2(0.785)}} = 0.228$$

and bolt tension with prying action is

$$P_{ba} = 64,152(1.228) = 78,788 \text{ lbs for pure axial load}$$

$$P_{bm} = 33,588(1.228) = 41,251 \text{ lbs for pure bending load}$$

The most probable load is the axial tension applied by the roof beam acting simultaneously with the ultimate moment capacity of the column. The bolts will also be subjected to a total shear equal to the column reaction. Because of the proximity of the connection to the fixed end, the connection is designed for the loads applied at that point. That is, $M = 738,936 \text{ in-lb}$ and $T = 64,524 \text{ lb}$.

From paragraph 5.6.5, the maximum resistance of the column is 59,512 lb, and the maximum horizontal load is 215,007 lbs (assuming only the quasi-static load component need be considered). The dynamic reaction of the column at its upper (fixed) end is (Table 5-3)

$$\begin{aligned} V &= 0.38R_m + 0.12F_v + M_{PS}/L \\ &= 0.38(59,512) + 0.12(215,007) + 738,936/149 \\ &= 53,375 \text{ lb} \end{aligned}$$

The shearing stress in the bolts is

$$f_v = \frac{53,375}{4(0.785)} = 16,998 \text{ psi}$$

The total bolt tension is

$$P_b = 1.228[P_{ba} + P_{bm}] = 1.228\left[\frac{64,524}{4} + 33,588\right] = 61,055 \text{ lb}$$

Reference 5-2 allows a bolt tensile stress of

$$\begin{aligned} F_t &= 1.7[70 - 1.6f_v] \leq 1.7(54) \\ &= 1.7[70 - 1.6(17.0)] \leq 91.8 \text{ ksi} \\ &= 72.8 \leq 91.8 \text{ ksi} \end{aligned}$$

Actual maximum tensile stress in the bolts is

$$F_t = \frac{61,055}{0.785} = 77,777 \text{ psi} = 77.7 \text{ ksi} > 72.8 \text{ ksi}$$

and the bolts appear somewhat overstressed but will not be changed at this time. Using an allowable tensile stress of 72.8 ksi, the capacity of each bolt is

$$P_b = 72.8 A_b = 72.8(0.785) = 57.2 \text{ kips}$$

The bending moment in the connection plate at the bolt line is

$$M = Qa = 0.228 \left[\frac{61,055}{4} + 33,588 \right] [2.0] = 22,276 \text{ in-lb}$$

Subtracting the 1.0625-inch hole from the plate, the allowable bending at the bolt line is

$$M_{all} = \frac{f_{dy}(p - 1.0625)t^2}{4} = \frac{39,600(3.0 - 1.0625)(1)^2}{4} \\ = 19,181 \text{ in-lb}$$

The bending moment in the plate at the face of the column is

$$M = T_b - Qa = \left[\frac{61,055}{4} + 33,588 \right] 1.5 - 22,276 = 51,002 \text{ in-lb}$$

Allowable bending at the column, assuming $p = 6$ inches, is

$$M_{all} = \frac{f_{dy}pt^2}{4} = \frac{39,600(6)(1.0)^2}{4} = 59,400 \text{ in-lb}$$

Therefore, bending of the plate is within the allowable limits at the column line but exceeded at the bolt line. In view of the conservatism of using end moments and shears in this analysis, the plate is considered satisfactory.

The shear load per bolt is

$$V = \frac{53,375}{4} = 13,344 \text{ lb}$$

The bearing stress is

$$f_b = \frac{V}{A_b} = \frac{13,344}{1(1)} = 13,344 \text{ psi}$$

The allowable bearing stress is

$$F_p = 1.35 f_{dy} = 1.35(39,600) = 53,460 \text{ psi}$$

5.6.8 Structural Response of Group 3 Suppressive Shield Steel Hoop

a. Given

The steel hoop of the Group 3 shield is located midway between the foundation slab and the roof slab. The hoop is placed around the outer circumference of the cylinder to support the S3x5.7 interlocking beams. It consists of ten continuous straps 5 inches wide by 1/2 inch thick to make a hoop cross section 5 inches wide by 5 inches thick.

b. Find

The response of the steel hoop using the blast loads given in illustrative example 5.6.1.

c. Solution

The first step is to determine the natural period of vibration of the hoop and that portion of the vertical beams it supports. The vertical beams are assumed to be fixed at the roof and foundation slab and supported at mid span by the steel hoop. The weight of the steel hoop is

$$W_R = \rho A = 0.283(25) = 7.075 \text{ lb/in}$$

From Table 5-3, one-third of the mass of the beam between supports is assumed concentrated at midspan for single degree of freedom analyses. It appears logical to assume the remaining two-thirds is distributed equally to each support. Since there are two beams bearing on the steel hoop, assume that 2/3 of the weight of a vertical beam is included in the weight of the steel hoop. The additional uniformly distributed weight contributed to the steel hoop by the beams is

$$W_B = 2/3 \frac{WLN}{2\pi R}$$

$$W_B = 2/3 \left(\frac{5.7}{12} \right) (60) \left(\frac{296}{2\pi(72.5)} \right) = 12.35 \text{ lb/in}$$

where

W = weight of beam per inch

L = length of beam, inches

N = total number of beams

R = radius to center of wall, inches

The total distributed steel hoop mass is

$$M = \frac{7.08 + 12.35}{386} = 0.0503 \text{ lb-sec}^2/\text{in}^2$$

The natural period of the ring is obtained from

$$T_N = 2\pi \left[\frac{mR^2}{EA} \right]^{1/2} = 6.28 \left[\frac{0.0503(72.5)^2}{29 \times 10^6 (25)} \right]^{1/2} = 0.0038 \text{ sec}$$

The maximum unit resistance of the ring beam is given by

$$r_m = \frac{f_{dy} A}{L_B R_i}$$

where

f_{dy} = dynamic yield strength, 39,600 psi

A = 25 in²

L_B = supported length of beam, 60 inches

R_i = inside radius of structure, 67.5 inches

Substituting,

$$r_m = \frac{39,600(25)}{60(67.5)} = 244.44 \text{ psi}$$

From example 5.6.1, the peak reflected pressure, P_r , is 3350 psi and its duration, t_1 , is 0.00025 seconds.

The peak quasi-static pressure, P_{qs} , is 187 psi and its duration, t_2 , is 0.066 seconds. The ratio of the durations of the two pulses to the period of the hoop are such that the structural response of the steel hoop is found from

$$\left[\frac{\frac{C_1 P_r}{r_m}}{\frac{T_N}{\pi t_1} \sqrt{2\mu-1}} \right]^2 + \frac{\frac{C_2 P_r}{r_m}}{\frac{T_N}{\pi t_2} \sqrt{2\mu-1} + \frac{1 - \frac{1}{2\mu}}{1 + 0.7 \frac{T_N}{t_2}}} = 1$$

where

$$r_m = 244.44 \text{ psi}$$

$$P_r = 3350 \text{ psi}$$

$$t_1 = 0.00025 \text{ sec}$$

$$t_2 = 0.066 \text{ sec}$$

$$C_1 = 0.944$$

$$C_2 = 0.056$$

$$T_N = 0.0038 \text{ sec}$$

A solution is obtained by trial and error and a ductility ratio, μ , of 15 was found to satisfy the equation. Substituting in the equation

$$\left[\frac{0.944(3350)}{244.44(25.987)} \right]^2 + \frac{0.056(3350)}{244.44(1.0277)} = 0.99 \approx 1.0$$

The circumferential stretching of the hoop at the elastic limit is

$$\Delta_L = \frac{f_{dy} A (2\pi R_R)}{AE} = \frac{39,600(25)(6.28)(72.5)}{25(29)(10)^6}$$

$$\Delta_L = 0.622 \text{ inch}$$

and the radial deflection is

$$\Delta_R = \frac{0.622}{2\pi} = 0.099 \text{ inch}$$

Since $\mu = X_m/X_y$, the maximum radial deflection of the hoop is

$$X_m = 15(0.099) = 1.485 \text{ inches}$$

5.7 LIST OF SYMBOLS

a	(1) Side of square plate (inches) (2) Plate dimension (inches)
A, A_1, A_2, \dots	Area (in^2)
A_s	Area of tensile steel (in^2)
A_v	Area of vertical web reinforcing (in^2)
A_{vH}	Area of horizontal web reinforcing over distance S_H (in^2)
b	(1) Width of cross section (inches) (2) Plate dimension (inches)
B	Peak total load (lbs)
C, C_1, C_2, C_3, \dots	Equation constants
d	(1) Effective depth of concrete member (inches)
d_w	Web depth (inches)
D	Diameter (inches)
E	Modulus of elasticity (psi)
f'_c	Static unconfined compressive strength of concrete (psi)
f'_{dc}	Dynamic compressive strength of concrete (psi)
f_{dy}	Dynamic tensile yield stress (psi)
f_y	Static tensile yield stress (psi)
F	Force (lbs)
F_{eq}	Force on equivalent single degree of freedom system (lbs)
F_t	(1) Total force (lbs) (2) Tensile load on bolt (lbs)
$F_t(t)$	Time varying force (lbs)
F_1, F_2, F_3, F_i	Ratios of $C_i B / R_m$ for given load characteristics and ductility ratio
H	Force per unit length (lbs)
i	Impulse (lbs-sec)
i_r	Reflected pressure impulse (psi-sec)
I	Moment of inertia of beam or moment of inertia of unit width of one-way slab (in^4)
I_a	Average of gross and cracked moment of inertia per unit width of concrete slabs (for short span in two-way slabs) or plastic section modulus of plate per unit width (in^4)

K, K_1, K_2, \dots	Spring constant (lb/inch)
K_e	Spring constant for elastic range (lb/inch)
K_{ep}	Spring constant for elastic-plastic range (lb/inch)
K_{eq}	Spring constant of equivalent single degree of freedom system (lb/inch)
K_E	Equivalent spring constant (lb/inch)
K_L	Load transformation factor
K_{LM}	Load-mass factor
K_M	Mass transformation factor
K_R	Resistance factor
K.E.	Kinetic energy (in-lb)
ℓ, L	Length (inches)
L_D	Deformed rebar required development length (inches)
m	Mass per unit length/area (lb-sec ² /in ²)
M	Mass (lb-sec ² /in)
M_b	Total mass of beam (lb-sec ² /in)
M_c	Total moment at critical section (inch/lb)
M_{eq}	Mass of equivalent single degree of freedom system (lb-sec ² /in)
M_p	Ultimate bending moment capacity (inch/lbs)
M_{pc}	Ultimate positive bending moment capacity per unit width at center of circular slab (in-lb/in)
M_{pfa}	Total ultimate positive bending moment capacity along midspan section parallel to short side, a (in-lb)
M_{pfb}	Total ultimate positive bending moment capacity along midspan section parallel to long side, b (in-lb)
M_{pm}	Ultimate bending moment capacity of beam at midspan (in-lb)
M_{ps}	Ultimate negative bending moment capacity per unit width at center of circular slab or ultimate bending moment capacity of beam at support (in-lb/in or in-lb)
M_{psa}	Total ultimate negative bending moment capacity along short edge, a (in-lb)

M_{Psa}^o	Ultimate negative bending moment capacity per unit width at center of edge a in direction of long span, b (in-lb/in)
M_{Psb}	Total ultimate negative bending moment capacity along long edge, b (in-lb)
M_{Psb}^o	Ultimate negative bending moment capacity per unit width at center of edge b in direction of short span, a (in-lb/in)
M_s	Total mass of spring (lb-sec ² /in)
M_t	Total mass (lb-sec ² /in)
P	(1) Pressure (psi) (2) Tensile reinforcing steel ratio
$p(t)$	Pressure as a function of time (psi)
P	Force (lbs)
r_m	Maximum unit resistance (psi)
R	(1) Radius (inches) (2) Resistance of element (lbs)
R_e	Elastic resistance (lbs)
R_i	Inside radius (inches)
R_m	Maximum resistance (lbs)
R_{meq}	Maximum resistance of equivalent single degree of freedom system (lbs)
R_o	Outside radius (inches)
s	Spacing of vertical web reinforcing (inches)
s_H	Vertical spacing of horizontal web reinforcing (inches)
S	(1) Section modulus (in ³) (2) Slope of strain hardening curve
t	Thickness (inches)
t_f	Flange thickness (inches)
t_m	Time of maximum displacement (inches)
t_o	Duration of positive pressure pulse (sec)
t_w	Web thickness (inches)
t_1, t_2, \dots	Pulse durations (sec)
T_N	Natural period of vibration (sec)
U	Strain energy (in-lb)
v_d	Direct shear stress (psi)

V	(1) Dynamic reaction at end or edge of symmetric element (lbs) (2) Total shear acting on section (lbs)
V_c	Ultimate shear force in concrete (lbs)
V_d	Total shear at support (lbs)
V_s	Shear capacity added by shear reinforcing (lbs)
V_u	Ultimate shear capacity (lbs)
V_y	Total shear acting on section (lbs)
V_A	Total dynamic reaction along one short edge, a (lbs)
V_B	Total dynamic reaction along one long edge, b (lbs)
V_1	Dynamic reaction at hinged end of nonsymmetric beams (lbs)
V_2	Dynamic reaction at fixed end of nonsymmetric beams (lbs)
W	Charge weight of explosive (lbs)
X	Displacement (inches)
\dot{X}	Velocity (in/sec)
\ddot{X}	Acceleration (in/sec ²)
X_e	Elastic limit displacement (inches)
X_{eq}	Displacement of equivalent single degree of freedom system (inches)
X_m	Maximum displacement (inches)
X_p	Elasto-plastic displacement (inches)
X_E	Equivalent elastic limit displacement (inches)
Z	(1) Scaled distance (ft/lb ³) (2) Plastic section modulus (in ³)
ϵ	Strain (in/in)
ϵ_{dy}	Dynamic yield strain (in/in)
μ	Ductility ratio
ν	Poisson's ratio
ρ	Mass density (lb-sec ² /in ⁴)
σ	Stress (psi)
σ_h	Hoop stress (psi)

σ_l	Longitudinal stress (psi)
σ_r	Radial stress (psi)
ω_N	Circular natural frequency (rad/sec)

5.7 REFERENCES

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